

Holographic superconductors from the massive gravity

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Abstract

A holographic superconductor is constructed in the background of a massive gravity theory. In the normal state without condensation, the conductivity exhibits a Drude peak that approaches a delta function in the massless gravity limit as studied by David Vegh. In the superconducting state, besides the infinite DC conductivity, the AC conductivity has Drude behavior at low frequency followed by a power law-fall. These results are in agreement with that found earlier by Horowitz and Santos, who studied a holographic superconductor with an implicit periodic potential beyond the probe limit. The results also agree with measurements on some cuprates.

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I. INTRODUCTION

As a new method to study strongly coupled field theory, the AdS/CFT correspondence[1] has been used to study the quantum many-body system in the past few years, which we call the AdS/CMT. A remarkable progress of AdS/CMT is the first holographic realization superconductivity[2], for review see[3]. However, in this setup the superconductor is homogeneous without broken translational symmetry, as a result of the momentum being not dissipate, the DC conductivity is infinite even in the normal metal phase of the superconductor. The translational symmetry breaking in AdS/CFT has been studied by the perturbative method and Drude(-like) peaks were discovered[4–8]. Some other related studies, including the construction of the inhomogeneous black hole solution and the corresponding holographic superconductor, by the perturbative method have also been obtained[9]. Furthermore, instead of the perturbative method, the authors in Refs.[10, 11, 13] constructed some periodic gravitational background to simulate the lattice by introducing a periodic scalar field or chemical potential on the boundary. With Einstein-DeTurck method, they solve the coupled partial equations numerically and the universal non-Drude frequency scalings with finite DC conductivity are disclosed. The first holographic setup of a lattice superconductor was studied by introducing a periodic charge density beyond probe limit in [12], and the expected infinite DC conductivity is also found in the superconducting state. Besides the way to solve partial differential equations with inhomogeneous sources as in[10–14], the use of a helical lattice, Q-lattice, or using spatially dependent but massless scalars can also enables us to break translation invariance while preserving homogeneity[15, 16]. We can also break translational symmetry spontaneously by introducing some topological terms or some addition field in the action[17]. At the same time, the first example of a modulated phase in string theory was found in[18]. Soon thereafter, they worked it out in D2-D8 model and also generalized the mechanism with magnetic fields and stuff both in D3-D7 and D2-D8 models [19].

Another mechanics to break translation symmetry is to introduce a mass term for the graviton as[20]

$$\mathcal{L}_I = \sqrt{-g}m^2\delta g_{tx}\delta g^{tx}, \quad (1)$$

which can be implemented in the massive gravity framework[21, 22]. Due to diffeomorphism breaking, the conservation of the stress-energy tensor in the massive gravity is broken.

However, the massive gravity suffer from the instability problem due to the introduction of the mass term for the graviton. To eliminate Boulware-Deser ghost, a non-linear massive gravity is constructed by introducing higher order interaction terms in the action[23]. In this framework, a charged black brane solution has been constructed and the conductivity has also been calculated[20]. A recent review of massive gravity can be found in [24].

In this paper, we are trying to construct a holographic superconductor in the massive gravity theory. In the normal metal phase state, by choosing proper values of the gravity mass we reproduced exactly the results in [20], both the Drude peak and power-law scaling behavior with the exponents γ equal to $-2/3$ of the AC conductivity. This power-law scaling are in agreement with measurements of the normal phase of bismuth-based cuprates[25] and also the results in[10, 11].

In the superconducting state, besides the infinite DC conductivity, the authors also found a Drude behavior at low frequency followed by a power law fall-off. These results are in striking agreement with measurements on cuprates [26] and also the implicit lattice results with condensation in[12]. However, in [12] the author found that the coefficient of the power law is temperature independent, this is different from the holographic superconductor from massive gravity, in which the the coefficient of the power law is temperature dependent.

The organization of the paper is as follows. In Section II we gives the model of the holographic superconductor and the superconducting phase transitions are also studied. In Section III the equations of motion for fluctuations are derived and the conductivity of the normal phase is reviewed. The conductivity of the superconducting state is given in Section IV. Finally, the discussions and conclusions are given in Section V.

II. SUPERCONDUCTING FROM NON-LINEAR MASSIVE GRAVITY

A. Model and equations

We start with the non-linear massive gravity action including a cosmological constant following [20]

$$S_G = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + \Lambda + m_g^2 \Sigma_{i=1}^4 c_i \mathcal{U}_i(g, f)] . \quad (2)$$

where $\Lambda = \frac{6}{L^2}$, f is the reference metric, c_i are constants and m_g is the mass of the graviton. \mathcal{U}_i are symmetric polynomials of the eigenvalues of the matrix $\mathcal{K}^\mu_\nu \equiv \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$

$$\begin{aligned}\mathcal{U}_1 &= [\mathcal{K}], \\ \mathcal{U}_2 &= [\mathcal{K}]^2 - [\mathcal{K}^2], \\ \mathcal{U}_3 &= [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3], \\ \mathcal{U}_4 &= [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4],\end{aligned}\tag{3}$$

where $[\mathcal{K}] \equiv \mathcal{K}^\mu_\mu$, $[\mathcal{K}^2] \equiv (\mathcal{K}^2)^\mu_\mu$ and $(\mathcal{K}^2)_{\mu\nu} \equiv \mathcal{K}_{\mu\alpha}\mathcal{K}^\alpha_\nu$ et. al.. As Ref.[20], we are interested in the case of a spatial reference metric (in the basis (t, z, x, y))

$$f_{\mu\nu} = (f_{sp})_{\mu\nu} = \text{diag}(0, 0, F^2, F^2),\tag{4}$$

where F is constant. In this case, there are only two spatial graviton mass terms ¹

$$m_g^2 \mathcal{U}_{sp} = m_g^2 (\alpha \mathcal{V}_1 + \beta \mathcal{V}_2),\tag{5}$$

where

$$\begin{aligned}\mathcal{V}_1 &= \mathcal{U}_1 = [\mathcal{K}] = F(\sqrt{g^{xx}} + \sqrt{g^{yy}}), \\ \mathcal{V}_2 &= \mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2] = 2F^2 \sqrt{g^{xx}g^{yy}},\end{aligned}\tag{6}$$

and we have denoted $\alpha = c_1$, $\beta = c_2$. Without loss of generality, we will set $F = 1$ in the following.

In order to build a holographic superconductor from non-linear massive gravity, we introduce a $U(1)$ gauge field and a charged complex scalar field, with action

$$S_M = -\frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(\frac{L^2}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} + |D\Psi|^2 + \frac{m_\Psi^2}{L^2} |\Psi|^2 \right),\tag{7}$$

where $\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \nabla_\mu - iqA_\mu$ with m_Ψ and q being the mass and charge of the complex scalar field, respectively. For convenience, we will set $L = 1$. From the non-linear massive gravity action (2) plus the matter action (7), one can obtain Einstein's

¹ The terms \mathcal{U}_3 and \mathcal{U}_4 vanish when we take the spatial reference metric (4).

equation, scalar equation and Maxwell's equation as, respectively

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{2}g_{\mu\nu}\Lambda + m_g^2 X_{\mu\nu} - \frac{1}{2}g^{\rho\sigma}\mathcal{F}_{\mu\rho}\mathcal{F}_{\nu\sigma} + \frac{1}{8}g_{\mu\nu}\mathcal{F}_{\rho\sigma}\mathcal{F}^{\rho\sigma} - \frac{1}{2}(D_\mu\Psi D_\nu^*\Psi^* + D_\nu\Psi D_\mu^*\Psi^*) + \frac{g_{\mu\nu}}{2}(m_\Psi^2|\Psi|^2 + |D\Psi|^2) = 0, \quad (8)$$

$$D_\mu D^\mu\Psi - m_\Psi^2\Psi = 0, \quad (9)$$

$$\nabla^\mu\mathcal{F}_{\mu\nu} - iq[\Psi^*D_\nu\Psi - \Psi D_\nu^*\Psi^*] = 0, \quad (10)$$

where

$$X_{\mu\nu} = \frac{\alpha}{2}(\mathcal{K}_{\mu\nu} - [\mathcal{K}]g_{\mu\nu}) - \beta\left((\mathcal{K}^2)_{\mu\nu} - [\mathcal{K}]\mathcal{K}_{\mu\nu} + \frac{1}{2}g_{\mu\nu}([\mathcal{K}]^2 - [\mathcal{K}^2])\right). \quad (11)$$

We take the metric ansatz

$$ds^2 = \frac{L^2}{z^2}\left(-g(z)e^{-\chi(z)}dt^2 + \frac{dz^2}{g(z)} + dx^2 + dy^2\right), \quad (12)$$

with

$$A = \Phi(z)dt, \quad \Psi = \Psi(z). \quad (13)$$

From the z component of Maxwell's equations, one finds that the phase of the complex scalar field Ψ must be constant so that we can set Ψ being real without loss of generality.

The equations of the scalar field and the Maxwell field are, respectively

$$\Psi'' + \left(\frac{g'}{g} - \frac{\chi'}{2} - \frac{2}{z}\right)\Psi' + \frac{q^2 e^\chi \Phi^2 \Psi}{g^2} - \frac{m_\Psi^2 \Psi}{z^2 g} = 0, \quad (14)$$

$$\Phi'' + \frac{\chi'\Phi'}{2} - \frac{2q^2 \Psi^2 \Phi}{z^2 g} = 0. \quad (15)$$

Einstein's equations are

$$\Psi'^2 - \frac{\chi'}{z} + \frac{q^2 e^\chi \Phi^2 \Psi^2}{g^2} = 0, \quad (16)$$

$$\frac{3}{z^2} - \frac{3}{z^2 g} - \frac{\alpha m_g^2}{zg} - \frac{\beta m_g^2}{g} + \frac{m_\Psi^2 \Psi^2}{2z^2 g} + \frac{q^2 e^\chi \Psi^2 \Phi^2}{2g^2} - \frac{g'}{zg} + \frac{e^\chi z^2 \Phi'^2}{4g} + \frac{1}{2}\Psi'^2 = 0, \quad (17)$$

At the horizon z_+ ($g(z_+) = 0$), by using a serious solution we can find that there are independent parameters

$$z_+, \Psi_+ \equiv \Psi(z_+), E_+ \equiv \Phi'(z_+), \chi_+ = \chi(z_+). \quad (18)$$

The scalar potential itself must go to zero at the horizon in order for the gauge connection to be regular. The Hawking temperature of the black solution is determined by the above quantities

$$T = \frac{1}{16\pi L^2}((12 + 4\Psi_+^2)e^{-\chi_+/2} - L^2 E_+^2 e^{\chi_+/2} + 4(\alpha + \beta)e^{\chi_+/2}). \quad (19)$$

By choosing a value of the scalar fields $m_\Psi^2 = -2$, the information of the boundary field theory can be read from the asymptotic behavior of the fields at boundary $z = 0$,

$$\Phi(z) = \mu - \rho z + \cdots, \quad (20)$$

$$\Psi(z) = \Psi_1 z - \Psi_2 z^2 + \cdots, \quad (21)$$

where the μ is the chemical potential and ρ is charge density on the boundary field theory. The order parameter of the holographic superconductor is expectation value of the scalar operator, it can be Ψ_1 or Ψ_2 depending on the quantization we choose. One is to set $\Psi_1 = 0$ while $\Psi_2 = \langle \mathcal{O}_2 \rangle$ is the order parameter. The other is to treat $\Psi_2 = 0$ as the source while $\Psi_1 = \langle \mathcal{O}_1 \rangle$ is the condensation value.

Before solving the four equations we can set both L and z_+ equal to one by using a scaling symmetry of the system. With fixed $z_+ = 1$ we still have three independent parameter at the horizon. We should also fix the value of $\chi_+ = 0$ at the horizon, then after a rescaling of time $t \rightarrow ct$ with $c = e^{\chi(z=0)}$ we can set $\chi = 0$ on the boundary. Then the Hawking temperature of the black hole is to be the temperature of the boundary field theory.

Finally, with the two independent parameter Ψ_+ and Φ_+ , we can solve the equations of motion by the shooting method after choosing a specific quantization for scalar field Ψ . We will see that after reducing temperature the system will enter a superconducting state.

B. the superconducting phase transition

We choose two combinations of α and β as examples followed [20], which are $\alpha = -1, \beta = 0$ and $\alpha = -0.75, \beta = 0$. The order parameter of $q = 1$ are shown in FIG.1 and FIG.2. Note that in this paper, we will set $m_g = 1$ throughout.

From the two figures we see that there indeed a critical temperature below that there are condensation for $\langle \mathcal{O}_2 \rangle$. However, the case of $\langle \mathcal{O}_1 \rangle$ is different, for the parameter $\alpha = -0.75, \beta = 0$ we even can not find a finite temperature solution with condensation, when $\alpha = -1, \beta = 0$, the condensation $\langle \mathcal{O}_1 \rangle$ takes a two valued behavior when $T/T_c < 0.95$,

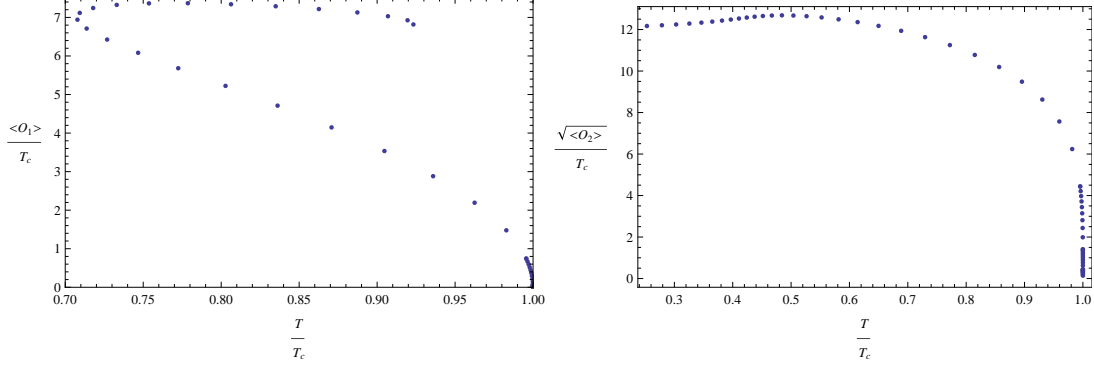


FIG. 1: The condensation phase diagram when $\alpha = -1, \beta = 0, q = 1$.

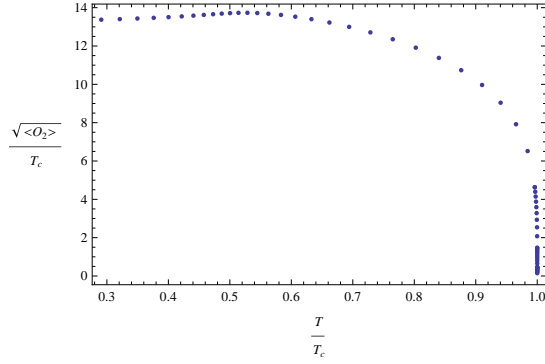


FIG. 2: The phase diagram when $\alpha = -0.75, \beta = 0, q = 1$.

which means that the phase transition structure is complex. However, the condensed region near the critical temperature is single valued, we choose this region when we compute the conductivity for the $\langle \mathcal{O}_1 \rangle$ case. Numerical fitting of the data close to T_c also indicates that the order parameter $(1 - T/T_c)^{1/2}$ at the critical point, which admit the mean field exponent.

III. CONDUCTIVITY OF NORMAL STATE

When there is no condensation, it has been confirmed in [20, 27] that for a region of α and β , the conductivity takes a Drude like peak at low frequency $\omega < T$, while $T < \omega < \mu$, the conductivity has a Power law scaling $|\sigma| \approx C + \frac{B}{\omega^{2/3}}$, C and B are determined by α and β . In [27], the author analytically computed the AC conductivity when the frequency is small and get exactly the Drude model form. In [28], a universal formula for the DC conductivity was derived based on the argument of [29] that the DC conductivity does not

vary with the radial coordinate, but depends only on geometric properties of the horizon. This formula derived in [28] relates the resistivity of the boundary field theory to the mass of the graviton evaluated on the horizon of the bulk black hole as

$$\sigma_{DC} = \frac{1}{q^2} \left[1 + \frac{q^2 \mu^2}{m^2(z_h)} \right], \quad (22)$$

in which $m^2(z_h) = -2\beta - \alpha/z_h$. By choosing the same value of α and β used in the previous section we reproduce both the Drude like scaling and the power-law scaling. The σ_{DC} can also vanish if we choose $q = 1, \mu = 1, \alpha = 0, \beta = 0.5$, this maybe a holographic realization of coherent metal in massive gravity.

A. Fluctuations and equations of motion

The conductivity is computed from the retarded Greens function of the transverse current via the Kubo formula

$$\sigma(\omega) = \frac{i}{\omega} G^R(\omega). \quad (23)$$

According to the *AdS/CFT* correspondence, the Green function $G^R(\omega)$ is computed by solving the equations of motion for fluctuations a_x and the fields coupled to it. No matter the temperature is below T_c or above T_c , there are three independent fluctuation fields a_x, h_{tx} and h_{zx} . The linear equations of the three fields are

$$a_x'' + \left(\frac{g'}{g} - \frac{\chi'}{2} \right) a_x' + \left(\frac{\omega^2 e^\chi}{g^2} - \frac{2q^2 \Psi^2}{z^2 g} \right) a_x + \frac{z^2 e^\chi \Phi'}{g} \left(\frac{2}{z} h_{tx} + h'_{tx} - i\omega h_{zx} \right) = 0, \quad (24)$$

$$\left(\frac{2}{z} h_{tx} + h'_{tx} - i\omega h_{zx} + \Phi' a_x \right) = - \frac{ie^{-\chi} g m^2(z)}{\omega} h_{zx}, \quad (25)$$

$$\left(\frac{2}{z} h_{tx} + h'_{tx} - i\omega h_{zx} + \Phi' a_x \right)' = \frac{m^2(z)}{g} h_{tx} - \frac{1}{2} \chi' \left(\frac{2}{z} h_{tx} + h'_{tx} - i\omega h_{zx} + \Phi' a_x \right). \quad (26)$$

in which $m^2(z) = -2\beta - \frac{\alpha}{z}$. If there is no condensation, the equations above will come back to the equations studied in [20, 27].

We can express h_{tx} as a function of h_{zx} ,

$$\frac{g}{m^2(z)} \left[- \frac{ie^{-\chi} g m^2(z)}{\omega} h_{zx} \right]' + \left(- \frac{ie^{-\chi} \chi' g^2}{2\omega} h_{zx} \right) = h_{tx}. \quad (27)$$

Finally we get two coupled non-linear equations for a_x and h_{zx} ,

$$a_x'' + \left(\frac{g'}{g} - \frac{\chi'}{2} \right) a_x' + \left(\frac{\omega^2 e^\chi}{g^2} - \frac{2q^2 \Psi^2}{z^2 g} \right) a_x + \frac{z^2 e^\chi \Phi'}{g} \left(-\Phi' a_x - \frac{ie^{-\chi} g m^2(z)}{\omega} h_{zx} \right) = 0, \quad (28)$$

$$\begin{aligned}
& \frac{2}{z} \left[\frac{g}{m^2(z)} \left(-\frac{ie^{-\chi} g m^2(z)}{\omega} h_{zx} \right)' + \left(-\frac{ie^{-\chi} \chi' g^2}{2\omega} h_{zx} \right) \right] \\
& + \left[\frac{g}{m^2(z)} \left(-\frac{ie^{-\chi} g m^2(z)}{\omega} h_{zx} \right)' \right]' + \left(-\frac{ie^{-\chi} \chi' g^2}{2\omega} h_{zx} \right)' \\
& -i\omega h_{zx} + \Phi' a_x = -\frac{ie^{-\chi} g m^2(z)}{\omega} h_{zx}.
\end{aligned} \tag{29}$$

The equations of motion is solved numerically by setting a ingoing boundary condition at the horizon that

$$a_x(z), h_{zx}(z) \propto (1-z)^{-\frac{i\omega}{4\pi T}}. \tag{30}$$

We set normalizable UV boundary conditions for the h_{zx} field. On the boundary, the behavior of the a_x is

$$a_x(z) = a_1 + a_2 z + \dots. \tag{31}$$

And then the retard green function is read as

$$G^R(\omega) = \frac{a_2}{a_1}, \tag{32}$$

Therefore, the conductivity can be expressed as

$$\sigma(\omega) = \frac{i}{\omega} \frac{a_2}{a_1}. \tag{33}$$

In the next subsection, we will present the results of conductivity with both Drude and power law scaling.

B. Drude like scaling and pwer law scaling

Following [20], we choose $\alpha = -1, \beta = 0$ and $\alpha = -0.75, \beta = 0$. In FIG.3 the conductivity of the both cases above are presented, the Drude peak is clear and the real part goes to one at large frequency. The Drude scaling at low frequency are shown in FIG.4 for three different temperature of both $\alpha = -1, \beta = 0$ and $\alpha = -0.75, \beta = 0$. When $\omega < T$ the data can be fitted well very well by the Drude formula

$$\sigma(\omega) = \frac{\sigma_{DC}}{1 - i\omega\tau}, \tag{34}$$

in which τ is the relaxation time.

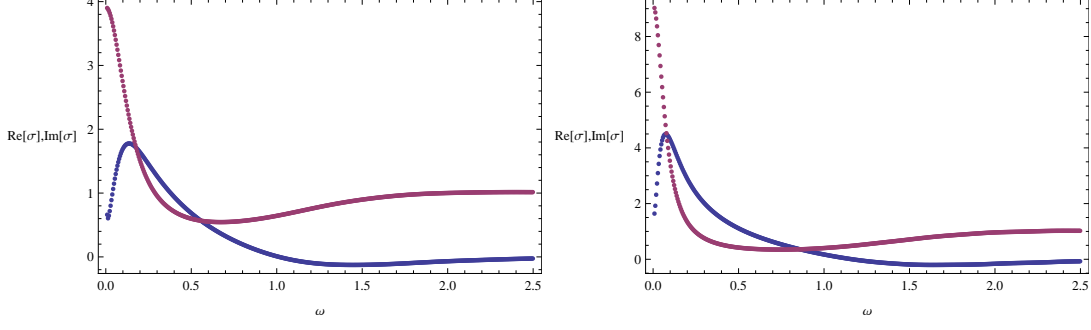


FIG. 3: Both real part and imaginary part of conductivity. Left: $\alpha = -1, \beta = 0, \mu = 1.732$. Right: $\alpha = -0.75, \beta = 0, \mu = 2.5$. We can see there is a Drude peak at low frequency.

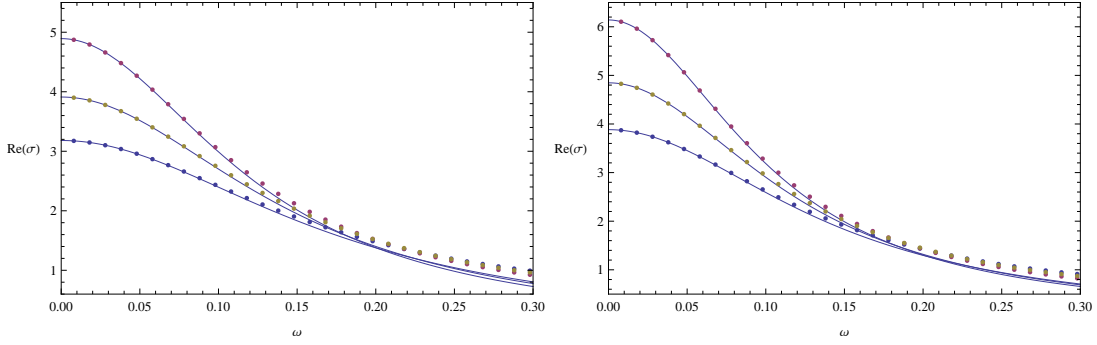


FIG. 4: Left: The Drude scaling for three different temperature $T = 0.08; 0.1; 0.114$ (from top to bottom) for $\alpha = -1, \beta = 0$. Right: The Drude scaling for three different temperature $T = 0.1; 0.119; 0.134$ (from top to bottom) for $\alpha = -0.75, \beta = 0$. When $\omega < T$, the curves can be fitted very well with the Drude scaling $\sigma(\omega) = \sigma_{DC}/(1 - i\omega\tau)$ when $\omega < T$. We also see that the σ_{DC} decrease when increasing the temperature and finally goes to one.

In the intermediate frequency the absolute value of the conductivity can be fitted well by the power law scaling

$$|\sigma(\omega)| = C + B\omega^{-\gamma}. \quad (35)$$

By a fine-tuning of the graviton mass, we can obtain $\gamma \approx 2/3$ (FIG.5), as claimed in the lattice models[10–13]. In addition, the results of $\alpha = -0.75, \beta = 0$ for three different temperature have also been presented in FIG.5, which indicate that the power law scaling hold when changing temperature. However, we must also point out that for a certain class of holographic lattice models, the so-called holographic *Q-lattice* model[16], the power law

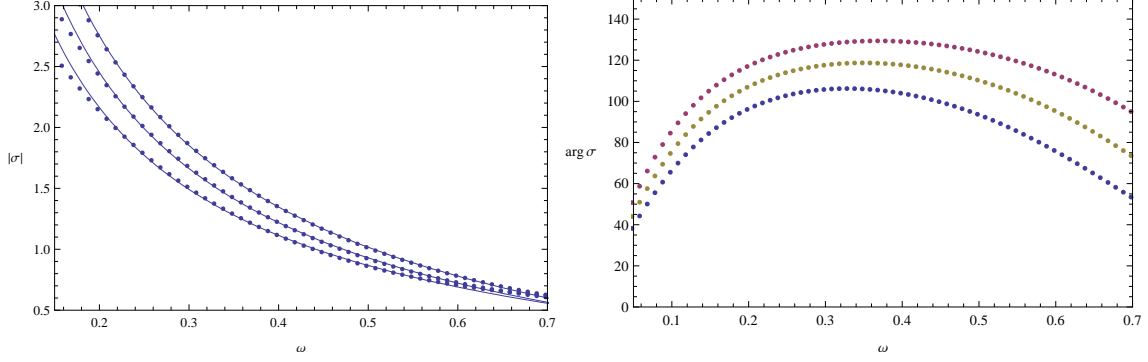


FIG. 5: Left: The power law scaling for three different temperature $T = 0.1; 0.119; 0.134$ (from top to bottom) for $\alpha = -0.75, \beta = 0$. The plots are fitted very well by $|\sigma(\omega)| = C + B\omega^{-2/3}$. Right: The phase of $\sigma(\omega)$.

scaling of $2/3$ exponent found in [10–12] is not universal, which may be depend on the specific model and need to be furthermore studied.

Both the Drude scaling at low frequency $\omega < T$ and Power law scaling at an intermediate regime $T < \omega < \mu$ shown above are in agreement with the results in [10, 11, 13] when an explicit inhomogeneous lattice is introduced in the bulk theory. There must be connections between the massive gravity theory and the inhomogeneous lattice models, in [30], by using the small-lattice expansion, the authors proved that, the presence of the latticed scalar source induces an effective mass for the graviton via a gravitational version of the Higgs mechanism.

IV. CONDUCTIVITY OF SUPERCONDUCTING STATE

In this section we are going to discuss the conductivity in the condensation state with non-zero Ψ . As a results of superconducting we see that the DC conductivity will become infinite. However, the finite frequency AC conductivity admit the same scaling behavior as the normal state as discussed in the previous section. Especially the power law scaling with $\gamma \approx -2/3$ are in agreement with the measurement in cuprates[25, 26].

At temperature is $T/T_c = 0.99$, the conductivity of the condensation phase with $\langle \mathcal{O}_1 \rangle$ of $\alpha = -1, \beta = 0$ are plotted in FIG.6. Compared the results of normal state in previous section, the difference is that the imaginary part of conductivity diverges at zero frequency.

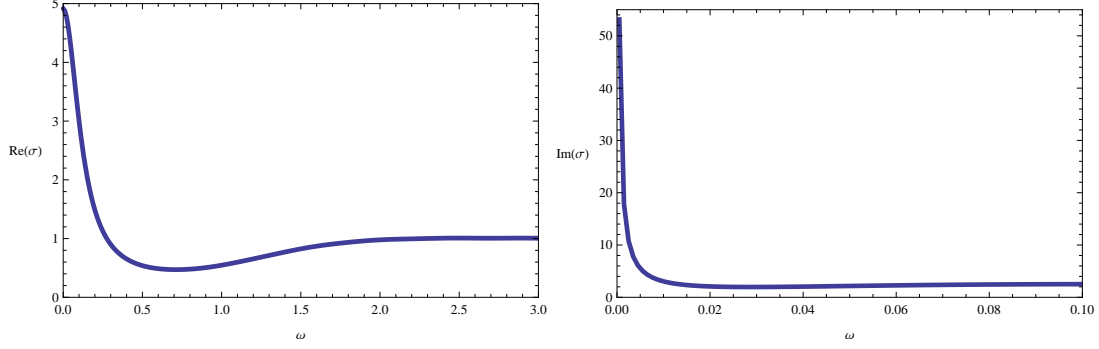


FIG. 6: The conductivity of the condensation phase with $\langle \mathcal{O}_1 \rangle$ for $\alpha = -1, \beta = 0$ and $T/T_c = 0.99$.

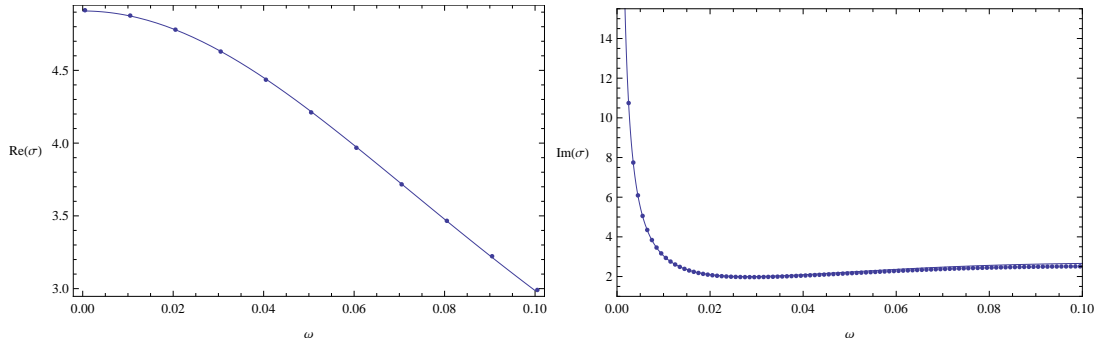


FIG. 7: At low frequency, the scaling of conductivity of the condensation phase with $\langle \mathcal{O}_1 \rangle$ for $\alpha = -1, \beta = 0$ and $T/T_c = 0.99$. The plots can be fitted very well by Eq.(36).

The divergence of imaginary part when $\omega = 0$ indicates that there is a delta function of the real part conductivity, which confirms the exist of superconducting state.

A. Low frequency region

When ω is small, we find that the conductivity can be fitted well by the formula as found in [12]

$$\sigma(\omega) = i \frac{\rho_s}{\omega} + \frac{\rho_n \tau}{1 - i\omega\tau}. \quad (36)$$

where ρ_s is the superfluid density and ρ_n is the normal fluid density, τ is the relaxation time.

In FIG.7 and FIG.8, we present the low frequency conductivity for both $\langle \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \rangle$ of $\alpha = -1, \beta = 0$. The agreement with Eq. (36) is very convincing. From FIG.8, we also see that when the temperature is lowered, the pole of the imaginary conductivity grows rapidly

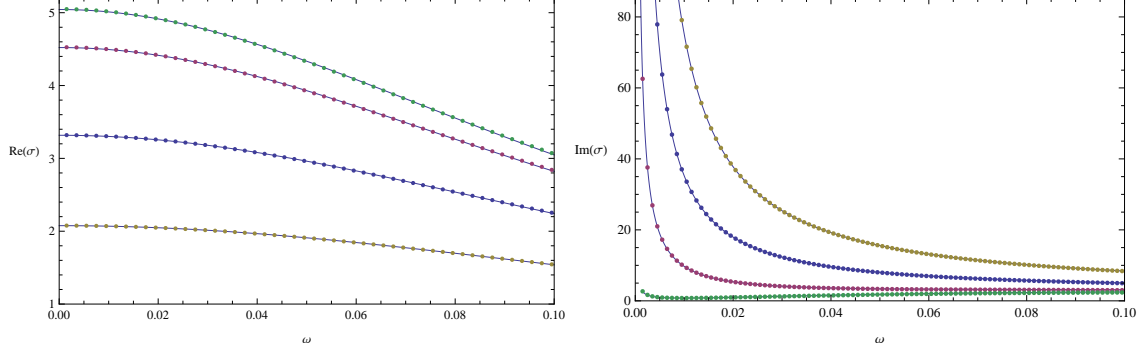


FIG. 8: $\alpha = -1, \beta = 0$, at low frequency, scaling of conductivity of the condensation phase with $\langle \mathcal{O}_2 \rangle$, the temperature from top to bottom is $T/T_c = 0.999; 0.981; 0.930; 0.856$. The data can be fitted very well by Eq.(36).

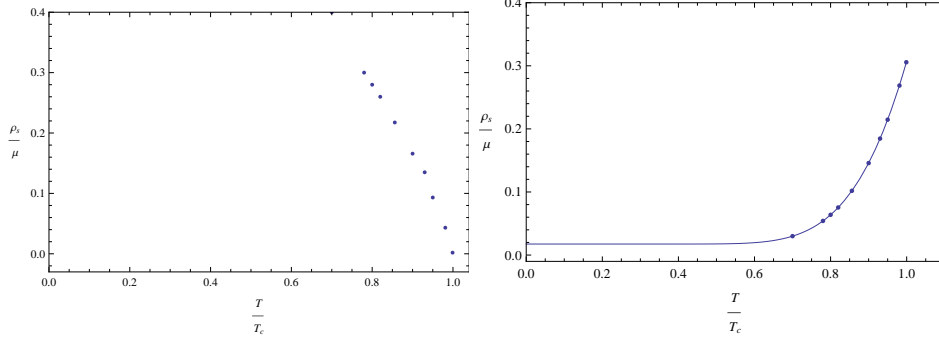


FIG. 9: The ρ_n and ρ_s in the superconducting state.

since there is more superfluid density ρ_s .

The temperature dependence of both ρ_n and ρ_s are plotted in FIG.9, we are working in the case of $\langle \mathcal{O}_2 \rangle$ when $\alpha = -1, \beta = 0$. The ρ_s is zero at T_c and increases rapidly when lowering temperature. The normal fluid density ρ_n can be fitted very well as

$$\rho_n(T) = a + be^{-\Delta/T}, \Delta \approx 6T_c. \quad (37)$$

This also similar to the BCS theory and also the result in [12]. The $\Delta \approx 6T_c$ is much larger than the value of a BCS superconductor with $\Delta \approx 1.7T_c$ means that the holographic superconductor in the massive gravity is indeed a strongly coupled superconductor. This is also a significant distinction that highlights how holographic lattice and massive gravity models generically lead to results that are qualitatively similar but quantitatively different.

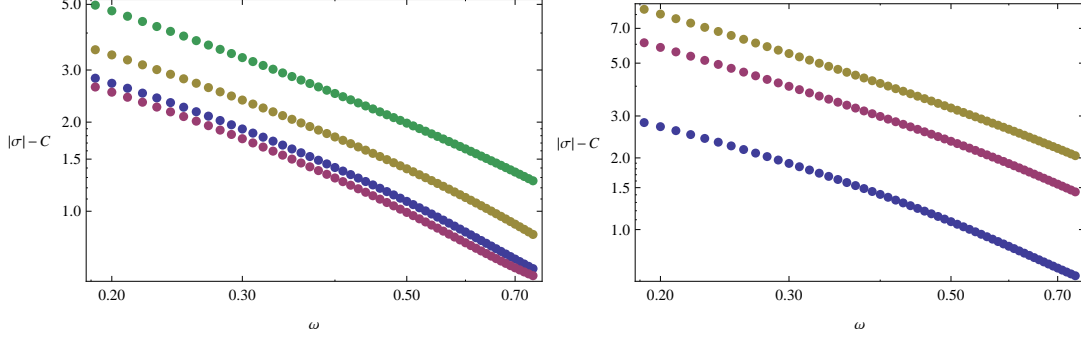


FIG. 10: The log-log plot of the normal part conductivity. Left: $\alpha = -1, \beta = 0$, the condensation phase with $\langle \mathcal{O}_2 \rangle$, the temperature from top to bottom is $T/T_c = 0.999; 0.981; 0.930; 0.856$. Right: from top to bottom are the cases: ($\alpha = -0.75, \beta = 0, \langle \mathcal{O}_2 \rangle > 0, T/T_c = 0.98$); ($\alpha = -1, \beta = 0, \langle \mathcal{O}_1 \rangle > 0, T/T_c = 0.98$) and ($\alpha = -1, \beta = 0, \langle \mathcal{O}_2 \rangle > 0, T/T_c = 0.856$).

B. Power law scaling and cuprates

At slightly larger frequency, the absolute value of the conductivity of the normal component follows the same power law Eq.(35) as was found in the normal phase above the critical temperature. In FIG. 10 we plotted the absolute value of the $\sigma(\omega)$ minus the off-set C vs frequency on a log-log plot after subtracting the part from superfluid density. In the left side of FIG. 10, the four lines corresponding to the same temperature as studied in FIG. 8 for the case $\alpha = -1, \beta = 0$ with condensation $\langle \mathcal{O}_2 \rangle$. While in the right side we plotted the results of both $\alpha = -1, \beta = 0$ and $\alpha = -0.75, \beta = 0$, and also both two kinds of condensation $\langle \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \rangle$. This clearly shows that the power law fall-off is completely unaffected by the condensation. Here, we also point out that we also need a fine-tuning of the graviton mass as that in the normal state.

The fact that the lines are parallel implies that the exponent is the same, which is in agreement with the results in [12] and also the experiment on bismuth-based cuprates [26]. The fact that the lines does not lie on top of each other implies that the coefficient B of the power law is temperature dependent, this is different from the results in [12], in which the author found the coefficient B of the power law is temperature independent.

V. CONCLUSION

In this paper, a holographic superconductor with momentum relaxation is constructed in the massive gravity proposed in [20]. In the normal state we reproduced the Drude scaling and power-law scaling as studied in [20, 27], while in the superconducting state, the superconducting part induces a delta function for real part conductivity when $\omega = 0$. However, the Drude scaling and power-law scaling of the normal part are not affected by the condensation. The results agree with the result in [12], in which the holographic superconductor with implicity lattice is studied. This agreement confirm that there is a deep connection between the massive gravity and the holographic lattice theory as studied in [30]. Recently, a holographic theory of strange metals is constructed from the Einstein-Maxwell-Dilaton massive gravity[31, 32], it would be interesting to find the superconductor/strange metal phase transitions in this setup. In this paper, we only focus on the cases of α and β that produce the normal metal state with Drude scaling, however, as pointed in [28], there are other possibility that the conductivity will show possible incoherent metal behaviors by choosing a proper mass term, this is also worth a further studying.

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